Math 16A Discussion Problem Set
Problems to be completed on week 15

1 December 2020

Group 1

Find the area between the curves: $x = 0$, $x = 4$, $y = \frac{1}{x+1}$, $y = \frac{x-1}{2}$.

Group 2

Suppose a company wants to introduce a new machine that will produce a rate of annual savings (in dollars) given by

$$S'(t) = 150 - t^2,$$

where t is the number of years of operation of the machine, while producing a rate of annual costs (in dollars) of

$$C'(t) = t^2 + \frac{11}{4}t.$$

1. For how many years will it be profitable to use this new machine?
2. What are the net total savings during the first year of use of the machine?
3. What are the net total savings over the entire period of use of the machine?

Group 3

Consider the function $f(x) = x(x^2 + 3)^7$.

1. Use the Fundamental Theorem of Calculus to evaluate $\int_{-5}^{5} f(x)dx$.
2. Use symmetry to describe how the integral from part (a) could be evaluated without using substitution or finding an antiderivative.

Group 4

An oil tanker is leaking oil at the rate given (in barrels per hour) by

$$L'(t) = \frac{80\ln(t+1)}{t+1},$$

where t is the time (in hours) after the tanker hits a hidden rock (when $t = 0$).

1. Find the total number of barrels that the ship will leak on the first day.
2. Find the total number of barrels that the ship will leak on the second day.
3. What is happening over the long run to the amount of oil leaked per day?

Group 5

The velocity v of the blood in a blood vessel was given as

$$v = k(R^2 - r^2),$$

where R is the (constant) radius of the blood vessel, r is the distance of the flowing blood from the center of the blood vessel, and k is a constant. Total blood flow (in millimeters per minute) is given by

$$Q(R) = \int_0^R 2\pi v r dr.$$

1. Find the general formula for Q in terms of R by evaluating the definite integral given above.
2. Evaluate $Q(0.4)$.

1 Extra Problems

1. You are given $\int_0^1 e^{x^2} dx = 1.46265$ and $\int_0^2 e^{x^2} dx = 16.45263$. Use this information to find
   (a) $\int_{-1}^1 e^{x^2} dx$
   (b) $\int_1^2 e^{x^2} dx$
2. The U.S. Census Bureau gives an age distribution that is approximately modeled (in millions) by the function

   $$f(x) = 40.2 + 3.50x - 0.897x^2$$

   where x varies from 0 to 9 decades. The population of a given age group can be found by integrating this function over the interval for that age group.

(a) Find the integral of $f(x)$ over the interval $[0, 9]$. What does this integral represent?

(b) Baby boomers are those born between 1945 and 1965, that is, those in the range of 4.5 to 6.5 decades in 2010. Estimate the number of baby boomers.